Topology and Dark Energy: Testing Gravity in Voids

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Modified gravity has garnered interest as a backstop against dark matter and dark energy (DE). As one possible modification, the graviton can become massive, which introduces a new scalar field. The new field can lead to a nontrivial equation of state (EOS) of DE which is density-and-scale-dependent. Tension between Type Ia supernovae and *Planck* can possibly be reduced. In voids the new scalar field dramatically alters the EOS of DE, gives rise to a significant gravitational slip between the two metric potentials, and also develops a topological defect (a domain wall) due to a nontrivial scalar field vacuum structure.

PACS numbers: 04.50.Kd, 95.36.+x, 98.62.Sb, 98.65.Dx

INTRODUCTION The non-detection of dark matter and the incomprehensibility of dark energy raises the anxiety level of theorists. Alternatives to Einstein's theory of general relativity (GR) is one strategy that is useful to have should the current dark sector crisis persist. In modifying gravity, the one lesson that has been learnt is the difficulty of finding an alternative theory that satisfies experimental constraints on all scales where gravity can be probed. High precision measurements on laboratory and solar system scales must accommodate astrophysical constraints on the two metric potentials in modified gravity theory from galactic to cosmological scales. Growth of density fluctuations and galaxy dynamics constrain the Newtonian potential. Light propagation constrains the sum of the two potentials. See [1–3] for review and citations.

Modifying gravity generally introduces a scalar that complements and couples to the tensor field of general relativity. The scalar must however be screened on small scales to be reconciled with the local precision tests of gravity. Three main mechanisms have been discussed for screening the new field. Chameleon [4] and f(R) theories [5] screen via density and decouple the scalar field at high density. Symmetron fields [6] weaken the scalar coupling at high density. Galileon/Braneworld fields [7, 8] and massive gravity (focus of this paper) operate via the Vainshtein effect [9], which decouples the scalar field in high-curvature regions.

Massive gravity (by assigning the graviton a mass of order the inverse Hubble scale) naturally and transparently restores coupling on the horizon scale, where one may then hope to account for the accelerated expansion of the universe [10]. The transition scale (i.e. the Vainshtein radius where decoupling occurs) is of order tens of Mpc today. This scale corresponds to the largest self-gravitating scales in the universe, and potentially the simplest to understand.

In galaxy clusters, the metric potentials are almost the same as with GR, which is not necessarily true in voids. The change in the potential is shown here to only be a few percent for over-densities (which has been found previously by [11–13]) but of order unity for voids. The apparent change in dynamics and lensing can be parameterized as an inhomogeneous (scale-dependent) EOS of DE, which could be measured with future galaxy surveys. Even more surprising, the scalar field forms a topological defect (domain wall) at the edge of voids.

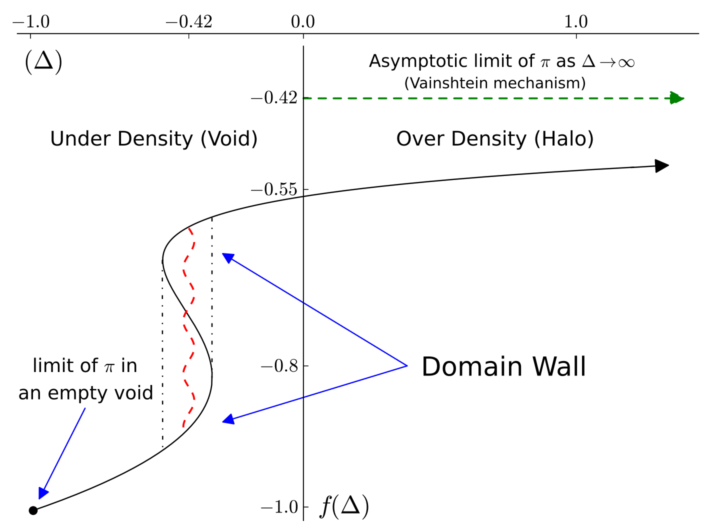

FIG. 1: Plot of f with $\pi'/r = f(\Delta)\Lambda^3$ where $\Delta = \delta\rho_{\rm m}/\rho_{m0}$ is the difference between the mean density inside of a radius r and the mean density today normalized by the mean density today.

MASSIVE GRAVITY Massive gravity (linearized) corresponds to a spin-2 field with five degrees of freedom, which can be decomposed into tensor, vector, and scalar modes. The effect of massive gravity in voids is well-characterized in the weak-field limit of GR and on sub-horizon scales, which is exactly the decoupling limit (see [7, 12, 14, 15] for a precise definition). In the decoupling limit, an effective theory characterizes massive gravity, which only depends upon the tensor and scalar degrees of freedom. The vector degrees of freedom decouple leaving a simple Lagrangian given by [10],

$$\mathcal{L} = h^{\mu\nu}\left(-\frac{1}{2}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + \frac{1}{\mathrm{m_{pl}}}\mathrm{T}_{\mu\nu} + \mathcal{L}(\pi)_{\mu\nu}\right). \qquad (1)$$

The Einstein tensor $(\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta})$ comes from the Einstein–Hilbert Lagrangian $(\mathrm{m^2_{pl}}\sqrt{-\mathrm{g}}\,\mathrm{R})$ assessed to the second order

in metric fluctuations $h^{\mu\nu}$ with $g_{\mu\nu} = \eta_{\mu\nu} + h^{\mu\nu}/\mathrm{m_{pl}}$, the Newton constant $8\pi\mathrm{G} \equiv \mathrm{m_{pl}^{-2}}$, $\eta_{\mu\nu} = (-1,1,1,1)$, and $\mathrm{m_{pl}}$ is the Planck mass. The second term of Eq. 1 is the standard coupling of GR to the stress-energy tensor of the system.

The third term is the Lagrangian for the scalar field π with $\mathcal{L}(\pi)_{\mu\nu} = \alpha X^{(1)}_{\mu\nu} + (\beta/\Lambda^3)X^{(2)}_{\mu\nu} + (\gamma/\Lambda^6)X^{(3)}_{\mu\nu}$, where α, β, and γ are arbitrary parameters of the theory but are taken to be $\mathcal{O}(1)$. Our effective scalar theory of π is valid up to the cutoff scale $\Lambda^3 = \mathrm{m_{pl}}m^2$, where m is the graviton mass. See [10, 15] for further details.

Ansatz We look for solutions which can accommodate both under and over-densities (halos and the solar system) and are consistent with cosmology. The equivalence principle implies that locally we can always define a metric (for all time) which will simply be Minkowski space plus a perturbation. The perturbation component will account for the non-trivial geometry of the space-time. Such coordinates are referred to as Fermi coordinates [7, 16]. In the Newtonian gauge,

$$ds^2 = -(1+2\Phi(r))dt^2 + (1-2\Psi(r))dx^2. \qquad (2)$$

which on small scales can be deduced from the general FRW metric by using the transformations $t_c = t + \mathrm{H}^2\vec{x}^2/2$ and $x_c = \vec{x}/a(1+1/4\mathrm{H}^2\vec{x}^2)$ between co-moving (t_c, x_c) and the local, physical Fermi coordinates (t, x). H is the Hubble parameter. a is the scale factor. We then add on local perturbations of the metric. Φ and Ψ encode both the background geometry and local metric perturbations. The standard stress–energy tensor (in Fermi coordinates) is simply $T_{\mu\nu} \simeq (\rho, \delta_{ij}p)$ where we have neglected off-diagonal terms ($\mathcal{O}(\mathrm{H}^2x^2) \sim v^2 \sim \mathrm{H}^2x^2v^2$ with velocity v). As with Ψ and Φ, ρ and p include local perturbations of matter and radiation relative to a background density. Finally, we make the ansatz that the field π (as well as the local metric and matter perturbations) are spherically symmetric and time-independent, $\pi = \pi(r)$.

By the least-action principle applied to Eq.1, we obtain two non-trivial Equations of Motion (EOM) for the metric and one EOM for π with $\prime = d/dr$

$$2\nabla^2\Psi = 8\pi G\langle\rho\rangle + \frac{6\,\alpha}{\mathrm{m_{pl}}}\frac{\pi'}{r} + \frac{6\,\beta}{\mathrm{m_{pl}}\Lambda^3}\frac{(\pi')^2}{r^2} + \frac{6\,\gamma}{\mathrm{m_{pl}}\Lambda^6}\frac{(\pi')^3}{r^3} \quad (3)$$

$$(\nabla^2 - \partial_i^2)(\Phi - \Psi) = 8\pi G\langle p\rangle - \frac{4\,\alpha}{\mathrm{m_{pl}}}\frac{\pi'}{r} - \frac{2\,\beta}{\mathrm{m_{pl}}\Lambda^3}\frac{(\pi')^2}{r^2} \quad (4)$$

$$\alpha\nabla^2(2\Psi - \Phi) + \frac{2\,\beta}{\Lambda^3}\frac{\pi'}{r}\nabla^2(\Psi - \Phi) - \frac{3\,\gamma}{\Lambda^6}\frac{(\pi')^2}{r^2}\nabla^2\Phi = 0 \quad (5)$$

where in Eq. 4 we are not summing over i but indexing over the spatial dimensions (x, y, z). Previous authors [12, 13] have found the above EOM, but have neglected the effect of pressure in Eq. 4. Also, $\langle\ \rangle$ gives the average value of a given quantity inside of a radius r. The quantities $\langle\rho\rangle$ and $\langle p\rangle$ are the average values of p and ρ inside of a radius r (with contributions from matter, DE, radiation, etc.) and $\langle\rho_{\mathrm{m}}\rangle = M(r)/(4/3\pi r^3)$ where $M(r) = \int \rho_{\mathrm{m}}(r')r'^2dr'd\Omega$ and $\rho_{\mathrm{m}}(r)$ is the density only in matter at a point r.

In linearized GR, perturbations and background evolution can be separated out by simply subtracting off the background evolution from the perturbed Einstein equations. Naïvely one would expect the same in massive gravity, but this proves impossible. Eq. 5 is non-linear in π and in the metric perturbations, which introduces cross-terms between the perturbed solution and the background solution. At zeroth order in π we look for solutions that accommodate both the background and local perturbations in the metric, density, and pressure.

The above EOM can be made dimensionless by multiplying by m^{-2} with $\Lambda^3 = \mathrm{m_{pl}}m^2$ where m is the graviton mass and then by setting $\pi'/r = f(\Delta)\,\Lambda^3$ where $\Delta = \delta\rho_{\mathrm{m}}/\rho_{m0}$ with ρ_{m0} as the matter density today and $\delta\rho_{\mathrm{m}}$ is the under-or-over-density of a void or a halo with $\delta\rho_{\mathrm{m}} = \langle\rho_{\mathrm{m}}\rangle - \rho_{m0}$ where $\delta\rho_{\mathrm{m}}$ and $\langle\rho_{\mathrm{m}}\rangle$ depend upon r. Hence, $\delta\rho_{\mathrm{m}}/\rho_{m0} = -1$ corresponds to an empty void as in devoid of matter but not DE. After making the appropriate substitutions, Eq. 5 becomes a quintic constraint equation for $f(\Delta)$ which depends upon the average pressure $\langle p\rangle$ and density $\langle\rho\rangle$ for a fixed radius r. Out of the five roots, three will typically be real.

Classes of Solutions The solutions for π' can be categorized into three separate classes. In a separate publication we will discuss in detail the various solutions, and only outline the different solutions here. Two of the classes have been discussed previously by [12, 13]. Neither of these cases produce interesting cosmological solutions. One class of solutions degravitates all mass. The second set of solutions generates an EOS of the universe which is equivalent to radiation at late times and generates negative densities.

The third class gives a new cosmological solution in which the EOS of DE generally tracks the energy density of matter, radiation, and curvature. There are several different sub-cases. In the first self-accelerating case (no cosmological constant), we can tune α, β, and γ to effectively generate a cosmological constant. The authors of [10] found a similar solution but with a different ansatz, which decoupled the evolution of the scalar field from the evolution of matter, radiation, and the metric. In a second self-accelerating solution (in a flat background), the EOS of DE will depend on the local density of matter. In a third self-accelerating case (with curvature $k \neq 0$), the scalar field can dynamically counteract the effect of curvature upon the expansion rate of the universe, which may appear as so-called "phantom" EOS of DE with $w < -1$.

The more general case with a cosmological constant will constitute the focus of the paper. In solving for the π field EOM, we search for roots of a quintic equation which has no general analytic solution. We search for all solutions numerically, which requires picking α, β, γ and m (graviton mass) such that densities are positive, the field π is ghost-free, and constraints on $\mathrm{H_0}$ and the Ω_{m} are satisfied. We will discuss the overall parameterization in a separate publication, but to be concise we will consider a single case with $\alpha = -0.45$, $\beta = -1.0$, and $\gamma = -0.86$ in which a topological feature appears (for other choices of parameters no domain wall exists).

The same solution exists for fixed ratios of γ^2/α and β^2/α. Finally to satisfy cosmological constraints, we have also set $8\pi G\rho_0/m^2 = 7.65$ where ρ_0 is the measured critical density today and have set $\Omega_{\rm m} = 0.3$ (the qualitative results presented are invariant under varying $\Omega_{\rm m}$ and $\rm H_0$).

In Fig. 1, we have plotted the solution of $\pi'/r\Lambda^3 = f(\Delta)$ as a function of the average under- or over-density $\delta\rho_{\rm m}$ interior to the radius r over the average density in matter today $\rho_{\rm m}$ with $\Delta = \delta\rho_{\rm m}/\rho_{\rm m}$. Fig. 1 shows how the field changes from the case of an under-density to an over-density.

Over-Density First, we consider a point mass $M > 0$ (as our over-density). In the limit of approaching the point mass $r \to 0$, then $\Delta \to \infty$ and $f(\Delta) \to -0.42$. The Vainshtein mechanism kicks in recovering GR ($\Phi \to \Psi \to -GM/r$). As $r \to \infty$ then we smoothly go onto the cosmological solution and $f(\Delta)$ takes on a value which gives the correct cosmological solution today with $2\Phi \to -(\dot{\rm H} + {\rm H}^2)\vec{x}^2$ and $2\Psi \to {\rm H}^2\vec{x}^2$ where H is the Hubble parameter. From the metric EOM, we then also recover the Friedmann equations.

Under-Density Far outside the void $\delta\rho_{\rm m} \simeq 0$ (the metric potentials match onto the cosmological solution found in the previous section). The average density $\langle\rho_{\rm m}\rangle$ drops as we move to the center of the void. Eventually at a certain radius, the derivative of the field must jump to a new branch (see Fig. 1) once $\delta\rho_{\rm m}$ drops to less than half of the present mass density of the universe today. The jump forces a discontinuity in the derivative of the field π' i.e. a kink, but in principle, we have the freedom to adjust the integration constant of π' such that π is continuous. Regardless, to match the derivative across the discontinuity requires the introduction of a wall.

The nature of the π wall is categorically different from a wall formed for a typical scalar field. A domain wall typically corresponds to a kink solution interpolating between two different vacuum states on different sides of the wall. The tension in the wall is due to the potential separating the two vacuum states in field space. The π field has no potential, $V(\pi) = 0$, but upon removing the static condition, the EOM for π becomes highly non-linear. As a parallel in fluid mechanics, non-linearities of the Navier–Stokes equation can lead to a discontinuity in density and pressure gradients (which we interpret as a shock wave). Similarly for π, a "shock wave" could also dynamically generate a discontinuity in π'. The presence of vacuum energy and matter are critical to induce the appearance of the domain wall. The π field in empty space will not induce topological structure (see [17] for a proof).

From Fig. 1, there exists a mapping of π'/r into real space, which effectively maps the vacuum of π into real space, since there is a one-to-one correspondence between π and π'. Clearly the mapping of π' is not one-to-one into real space, but we speculate that in a higher-dimensional space such a problem might be avoided. We can amend this problem by noting that a quintic equation corresponds to a curve on a Riemann surface which has multiple branch points. In fact, f loops around two branch points producing the multivalued function in real space in Fig. 1. On the Riemann surface, f is single valued! There exists a one-to-one mapping between real space

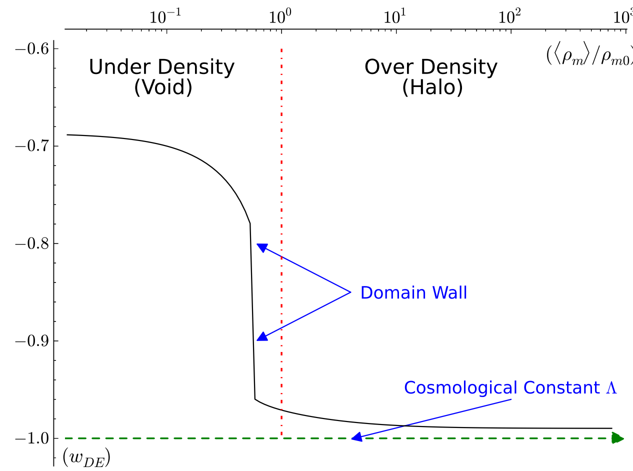

FIG. 2: Plot of EOS of DE $w_{\rm DE}$ where $\langle\rho_{\rm m}\rangle/\rho_{\rm m0}$ is the mean density of matter interior to a radius r over the mean density today $\rho_{\rm m0}$.

and π'/r for f on the Riemann surface. If we allow a complex extension of Δ and π', the quintic equation corresponds to hyper-elliptic surfaces which are compact Riemann surfaces with a genus greater than one which we identify as the vacuum of π by analogy with a standard Nambu–Goldstone Boson (NGB).

By the above identification, we can make sense of the appearance of a domain wall. The π field can be interpreted as a NGB (see [18]). Frequently the topologically non-trivial vacuum of a NGB when mapped into real space leads to the appearance of topological structures. Similarly, our function f due to its wrapping around branch points is equivalent to a non-trivial wrapping around the hyper-elliptic surface. Further properties of the domain wall may potentially be inferred from the Riemann surface.

Dynamical Dark Energy Massive gravity effects DE. The mean density inside a sphere of radius r is $\langle\rho(r)\rangle = \langle\rho_{\rm m}(r)\rangle + \rho_{\rm cc}$ where $\rho_{\rm m}$ is the cosmological density of matter today. $\rho_{\rm cc}$ is the energy density from the cosmological constant. Massive gravity also contributes to the energy density of the universe ($\rho_{\rm mg}$) via the last terms of Eq. 3. Similarly in Eq. 4 $\langle p\rangle = -\rho_{\rm cc}$ neglecting radiation. Massive gravity also adds to the pressure of the universe $p_{\rm mg}$ via the last terms in Eq. 4.

Massive gravity naturally leads to inhomogeneous DE evolving in time. In Fig. 1, the π field responds to changes in the local density. As π evolves, so does $\rho_{\rm mg}$ and $p_{\rm mg}$. The EOS of DE is $w_{\rm DE} = \frac{\rho_{\rm cc}+\rho_{\rm mg}}{\langle p\rangle + p_{\rm mg}}$. We have plotted $w_{\rm DE}$ in Fig. 2 as a function of the average density over the mean density today. Today (outside of halos and voids) $w_{\rm DE} = -0.97$. As we move into a halo, $w_{\rm DE}$ asymptotes towards -1. Conversely, at the center of a very under-dense void, $w_{\rm DE}$ jumps to -0.69. We can also read Fig. 2 as a measure of how the EOS of DE changes with redshift. At high redshift, the average density of the universe becomes large compared to the mean density today, equivalent to taking $\langle\rho_{\rm m}\rangle/\rho_{\rm m0} \to \infty$. At early times, the scalar field becomes unimportant which is merely a man-

ifestation of the Vainshtein mechanism. In this epoch, DE is then unimportant for the evolution of the universe.

Recent cosmic microwave background results from the *Planck* satellite [19] indicate (with 2.5σ significance) a nearly 10% lower value of the Hubble constant (H_0) compared to values found from local-Universe and supernova-based measurements, e.g. [20]. Although a number of SNIa systematics and concordance-model uncertainties could account for this difference, such an effect could also be explained within our model by a significant fraction of low-density void in the local Universe or along the line of sight. For a difference Δw in the background EOS, $\Delta\mathrm{H}/\mathrm{H}_0 \approx 3z\Delta w/2$. With a difference Δw driven by $w_{\mathrm{DE}} \approx -0.7$ in low-density voids and $w_{\mathrm{DE}} \approx -1$ elsewhere, a higher value of H_0 would be inferred than from the cosmic microwave background. The required local density of $< \rho_{\mathrm{m0}}/2$ appears consistent with recent observational studies, e.g. [21, 22]. This could potentially relieve the tension between the different Hubble constant estimates up to the observed 10% level.

CONCLUSION Massive gravity can naturally give rise to an inhomogenous EOS of DE, which will be dependent upon local matter density and scale. The scale-dependent EOS of dark energy may reduce the tension between relatively local measurements of dark energy from SNIa and global measurements from the CMB. Detailed analysis of voids may be the most direct way to infer the existence of a scale-dependent and density-dependent EOS of DE since perturbations in matter will lead to perturbations in DE. We can infer the existence of inhomogeneous DE via dynamics and lensing. The motion of galaxies from the geodesic equation constrains Φ (in the weak-field limit). However, for massless particles, the geodesic equation (lensing) depends upon the lensing potential $\Phi_{\mathrm{L}} = 1/2(\Phi+\Psi)$. We can infer the amount of material in and around a void by simply counting galaxies. It is then possible to infer the EOS of DE in a void and test the predicted large gravitational slip between the metric potentials, which should be achievable in future surveys [23, 24].

The EOS of DE will change the growth rate of voids. The ISW effect can also be a good way to constrain the EOS of DE in a void [25]. The domain wall may also directly affect lensing, dynamics of galaxies and cosmic rays crossing the domain wall. With a magnetic field, charged particles could be accelerated via an interaction with the π field, and as with axions/pseudo-Nambu–Goldstone bosons, distinct photon conversion/polarization could take place.

Dimensionally, the tension in π goes like $\tau \sim \Lambda^3$. Matching the metric potentials across the the discontinuity in π' will give a precise solution for the tension in the domain wall. We should also be able to calculate the wall tension via non-linear field dynamics and via an analysis of the vacuum structure of π. GR implies that they should all give the same solution.

Finally, the non-renormalization theorem for galileons [26] appears mysterious. The authors of [18] showed that the terms in $\mathcal{L}(\pi)$ are in fact Wess–Zumino–Witten (WZW) terms. If indeed our solutions for π are 'onto', then the non-renormalization theorem may be a consequence of the compact vacuum of π, as was found in the original WZW model.

ACKNOWLEDGEMENTS We would like to thank J. Khoury in particular for his extensive discussion and help. In addition, we would also like to thank P. Ferreira, W. Hu, M. Kamionkowski, E. Mörtsell, S. Sjörs, and M. Wyman for discussions and acknowledge the support of the Templeton Foundation for MS and a grant from the ERC for JS and DS.

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